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Minijet transverse-energy production in the next-to-leading order in hadron and nuclear collisions

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Abstract. The transverse-energy flow generated by minijets in hadron and nuclear collisions into a given rapidity window in the central region is calculated in the next-to-leading-order (NLO) in QCD at RHIC and LHC energies. The NLO transverse-energy production in pp collision cross sections is larger than that in the leading-order (LO) ones by the factors of $K_{\mathrm{RHIC}} \sim 1.9$ and $K_{\mathrm{LHC}} \sim 2.1$ at RHIC and LHC energies, respectively. These results were then used to calculate the transverse-energy spectrum in nuclear collisions in a Glauber geometrical model. We show that accounting for NLO corrections in the elementary pp collisions leads to a substantial broadening of the $E_{\perp}$ distribution for the nuclear ones, while its form remains practically unchanged.

1 Introduction

Minijet physics is one of the most promising applications of perturbative QCD to the analysis of processes with multiparticle production. The minijet approach is based on the fact that some portion of transverse energy is produced in the semihard form (i.e., it is perturbatively calculable because of the relatively large transverse momenta involved in the scattering) but is not observed in the form of customary hard jets well separated from the soft background. A notable feature of this approach is a predicted rapid growth of the integrated perturbative cross section of parton–parton scattering, responsible for perturbative transverse-energy production, with energy. At RHIC (200 GeV/A in CMS) and especially LHC (5500 GeV/A) energies, the perturbative cross section becomes quite large and in fact even exceeds the inelastic and total cross sections for sufficiently large rapidity intervals. The crossover from the regime described by conventional leading twist QCD and the one where multiple hard interactions are important is one of the most important problems of the minijet approach [1]. The field is actively developing; recent reviews on the subject containing a large number of references are, e.g., [2] and [3].

Minijet physics for ultrarelativistic heavy ion collisions has special importance because minijets with large enough transverse momenta are produced at a very early stage of the collision, thus forming an initial parton system that can further evolve kinetically, or even hydrodynamically, so that the minijet physics describes the initial conditions for subsequent collective evolution of parton matter [4–7] (see also a recent review [8]).

Among the recent developments, there is a new approach to minijet production based on the quasi-classical treatment of nuclear gluon distributions [9–13] and a description based on the parton cascade approach [14].

The perspective of having a perturbatively controllable description of a substantial part of the inelastic cross section is certainly very exciting[1]. However, if the accuracy of the predictions given by minijet physics is to be determined, the existing calculations have to be expanded in several directions.

In this paper, we discuss the conceptually simplest extension of the leading-order (LO) calculation of the transverse-energy spectrum produced in heavy ion collisions presented in [6,16,17] by including the next-to-leading-order (NLO) contributions to this cross section. The NLO corrections to a conventional high $p_{\perp}$ jet production cross section were computed in [18–20] for Tevatron energies. Later, the code of [19] was used for calculating this cross section for RHIC and LHC energies in [21]. The necessity of doing this computation in the minijet region was, of course, clearly understood and emphasized in the literature on minijet physics [3,22]. A clear goal here is to establish an accuracy of the LO prediction by explicitly computing the NLO corrections to it.

The outline of the paper is as follows.

In the second section, we describe a calculation of the next-to-leading-order (NLO) cross section of transverse-energy production in proton–proton collisions using the Monte Carlo code developed by Z. Kunzst and D. Soper [19] and study a deviation from the LO result.

In the third section, the computed NLO cross section is used in the calculation of transverse-energy production in heavy ion collisions, where the nuclear collision is de-

[1] The mechanism responsible for the growth of the inelastic cross section, as such, can be soft; see [1] and the recent analysis [15]

scribed as a superposition of the nucleon–nucleon collisions in a Glauber geometrical approach of [6]. We show that NLO corrections lead to a substantial broadening of the transverse-energy spectrum.

In the last section, we discuss the results and formulate the conclusions.

2 NLO minijet transverse-energy production in hadron collisions

The basic difference of minijet physics from that of the usual high $p_\perp$ jets is that the minijets can not be observed as jets as such. Experimentally, they manifest themselves in more inclusive quantities, such as the energy flow into a given rapidity window. The NLO calculation of a jet cross section contains a so-called jet-defining algorithm specifying the resolution for the jet to be observed, e.g., the angular size of the jet-defining cone; see, e.g., [23]. Since a minijet can not be observed as an energy flow into a cone separated from the rest of the particles produced in the collision, some of the restrictions employed in the standard definition of a jet should be relaxed. A natural idea is to define a minijet-produced "jet" as a transverse energy deposited in some (central) rapidity window and a full azimuth. This would require a standard detector setup for studying the central rapidity region in nuclear collisions at RHIC and LHC.

The inclusive cross section is obtained by the integration of the differential one over the phase space on the surface corresponding to the variable in question. Schematically, the NLO distribution of the transverse energy produced into a given rapidity interval $y_a < y < y_b$ is equal to

$$\begin{aligned}\frac{\mathrm{d}\sigma}{\mathrm{d}E_\perp} = &\int D^2PS\frac{\mathrm{d}\sigma}{\mathrm{d}^4p_1\mathrm{d}^4p_2}\\ &\times\delta\left(E_\perp - \sum_{i=1}^{2}|p_{\perp i}|\theta(y_{\min} < y_i < y_{\max})\right)\\ &+\int D^3PS\frac{\mathrm{d}\sigma}{\mathrm{d}^4p_1\mathrm{d}^4p_2\mathrm{d}^4p_3}\\ &\times\delta\left(E_\perp - \sum_{i=1}^{3}|p_{\perp i}|\theta(y_{\min} < y_i < y_{\max})\right), \quad (1)\end{aligned}$$

where the first contribution corresponds to the two-particle final state and the second contribution to the three-particle one. The two-particle contribution has to be computed with one-loop corrections taken into account.

As in all NLO calculations, the separation between the real emission and virtual exchange requires regularization, in addition to the usual ultraviolet renormalization, of the infrared and collinear divergences. Physically, this means that a distinction between a two-particle and three-particle final state becomes purely conventional.

In perturbative QCD, one can meaningfully compute only infrared stable quantities [26], in which the divergences originating from real and virtual gluon emission

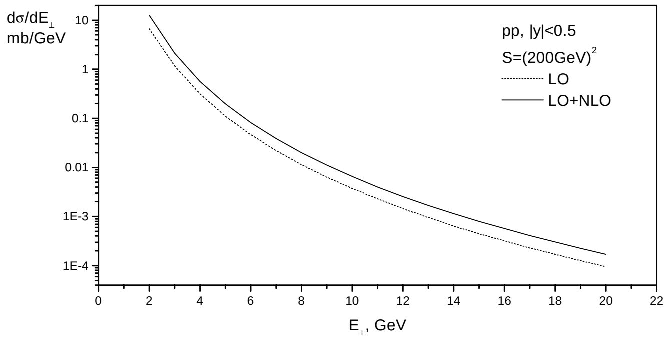

Fig. 1. NLO (solid line) and LO (dashed line) transverse-energy spectrum in a unit central rapidity window for pp collisions at RHIC energy $\sqrt{s} = 200$ GeV

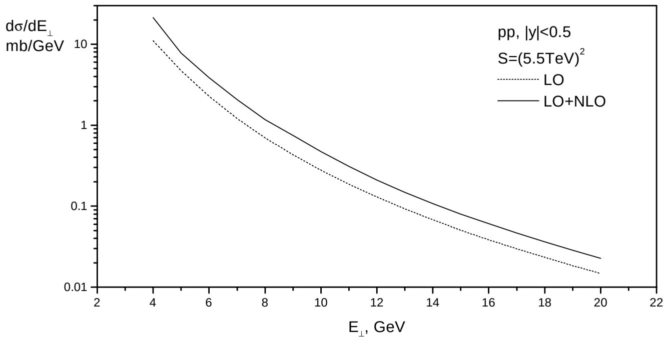

Fig. 2. NLO (solid line) and LO (dashed line) transverse-energy spectrum in a unit central rapidity window for pp collisions at LHC energy $\sqrt{s} = 5500$ GeV

cancel each other out, so that the addition of very soft gluons does not change the answer. It is easy to convince oneself that the transverse-energy distribution into a given rapidity interval (1) satisfies the above requirement.

In more physical terms, the main difference between the LO and NLO calculations is that unlike in the LO case, the number of produced (mini)jets is no longer an infrared-stable quantity in the NLO computation, i.e., it can not be predicted. For example, we can no longer calculate the probability of the acceptance window being hit by a given number of minijets, which is one of the important issues in an event-by-event analysis of the initially produced gluon system (for details, see [24]). In view of the applications of the NLO results for nucleon–nucleon collisions to the nuclear ones, this means, in turn, that the elementary block in the geometric approach no longer describes the production of a fixed number of jets (two jets in the LO case), but rather the production of transverse energy into a kinematical domain specified by the jet defining algorithm.

The calculation of the transverse-energy spectrum was performed using the Monte Carlo code developed by Kunzst and Soper [19], with a jet definition appropriate for the transverse-energy production specified in (1). The results for the cross section of transverse-energy production into a central rapidity window $-0.5 \leq y \leq 0.5$ are presented in Fig. 1 for RHIC energy ($\sqrt{s} = 200$ GeV) and Fig. 2

Table 1.

$\sqrt{S}$, GeV	LO/NLO	α	a, mb/GeV	$\sigma(E_0)$, mb	$\sigma_{\rm in}$, mb	E_0^*, GeV
5500	LO	4.14	3.7×10^3	15.3	66.3	2.5
5500	LO+NLO	4.24	7.8×10^3	26.9	66.3	3.0
200	LO	4.91	2.7×10^2	0.31	41.8	1.1
200	LO+NLO	4.92	5.0×10^2	0.55	41.8	1.3

for the LHC one ($\sqrt{s} = 5500$ GeV), where for LHC, we have chosen the energy of proton–proton collisions, which is available for protons in the lead nuclei in PbPb collisions. The calculations were performed by the use of the MRSG(-) structure functions [25]. The parameters for the fits for the spectra having the functional form $a * E_\perp^{-\alpha}$ are given in Table 1.

We see that, taking into account the NLO corrections, transverse-energy production can roughly be described by the introduction of effective K factors $K_{\rm RHIC} \sim 1.9$ and $K_{\rm LHC} \sim 2.1$. This agrees well with the "expected" K factor used in earlier publications [2,3]. Let us note that while at RHIC energies, the slopes of the LO and NLO curves are practically the same, at LHC energies, the NLO slope is about 2% larger than the LO one.

In the third column, we give the values of the integrated minijet production cross section

$$\sigma(E_0) = \int_{E_0}^{\infty} {\rm d}E_\perp \, \frac{{\rm d}\sigma}{{\rm d}E_\perp} \tag{2}$$

for the cutoff value $E_0 = 4$ GeV. The range of validity of a leading twist approximation for transverse-energy production in any given rapidity window is determined by the inequality

$$\sigma(E_0) = \int_{E_0}^{\infty} {\rm d}E_\perp \, \frac{{\rm d}\sigma}{{\rm d}E_\perp} \leq \sigma_{\rm inel} \tag{3}$$

The equality in the above formula fixes the limiting value of the cutoff E_0^*. The values of the inelastic cross section[2] are shown in the fourth column of Table 1, and the limiting cutoff values E_0^* are shown in the fifth one. We see that the limiting values E_0^* are quite small. We stress that the values of E_0^* depend rather strongly on the rapidity window under consideration. The limiting cutoff values shown in Table 1 refer to the central unit rapidity interval, and thus present a lower bound with respect to those corresponding to larger rapidity intervals.

Let us also note that, as mentioned before, we had to fix a scale for the NLO logarithmic corrections, which for the above calculations was chosen to be $\mu = E_\perp$. We have checked that variations of this scale in the range $0.5\ E_\perp \leq \mu \leq 1.5\ E_\perp$ do not produce significant variations of the result; so the NLO calculation is stable and thus produces a reliable prediction for the difference between the LO and NLO cases.

[2] The inelastic cross section is computed through the use of the parametrization $\sigma_{\rm inel}(s) = \sigma_0 * (s/s_0)^{0.0845} * (0.96 - 0.03 * \log(s/s_0))$, where $s_0 = 1$ GeV, $\sigma_0 = 21.4$ mb; this gives a good description of the existing experimental data [27]

3 NLO transverse-energy production in nuclear collisions

In this section, we turn to an estimate of the transverse-energy production in nuclear collisions from the Glauber-type approach, in which these collisions are considered to be an impact parameter-averaged superposition of hadron–hadron collisions. We shall follow the geometrical approach similar to that of [6] and consider the Gaussian approximation to the transverse-energy distribution at a given impact parameter in the collision of two nuclei with atomic numbers A and B:

$$\frac{{\rm d}\omega_{AB}}{{\rm d}E_\perp} = \frac{1}{\sqrt{2\pi D_{AB}}} \exp\left(-\frac{(E_\perp - \langle E_\perp\rangle_{AB}(b))^2}{2D_{AB}(b)}\right), \tag{4}$$

where $\langle E_\perp\rangle_{AB}(b)$ is a mean transverse energy produced at a given impact parameter b:

$$\langle E_\perp\rangle_{AB}(b) = ABP_{AB}(b)\langle E_\perp\rangle_{pp}(b). \tag{5}$$

D_{AB} is a dispersion of the $E_\perp$ distribution given by

$$D_{AB}(b) = ABP_{AB}(b)\left(\langle E_\perp^2\rangle_{pp} - P_{AB}(b)\langle E_\perp\rangle_{pp}^2\right), \tag{6}$$

and P_{AB} is the probability of nuclear scattering at a given impact parameter, and the normalization is fixed by the inelastic cross section for minijet production in pp collisions (2). Let us note that the Glauber model we use is similar to that of [6] in that the transverse-energy distribution at a given value of the impact parameter is approximated with the Gaussian (see (4)), but differs from the Gaussian in its use of the Bernoulli process instead of the Poisson one in [6], as well as in its different overall normalization at the integrated minijet production cross section in the rapidity window under consideration. The final expression for the cross section of transverse-energy production in nuclear collisions is obtained from (4) by integration over the impact parameter:

$$\frac{{\rm d}\sigma}{{\rm d}E_\perp} = \int {\rm d}^2 b \, \frac{{\rm d}\omega_{AB}}{{\rm d}E_\perp} \tag{7}$$

The resulting transverse-energy distributions are plotted in Fig. 3 and Fig. 4 for PbPb collisions for RHIC and LHC energies, respectively. We see that the main effect of taking into account the NLO corrections is a considerable broadening of the shoulder of the distribution, which basically follows from the increase in the magnitude of the transverse-energy production cross section from LO to

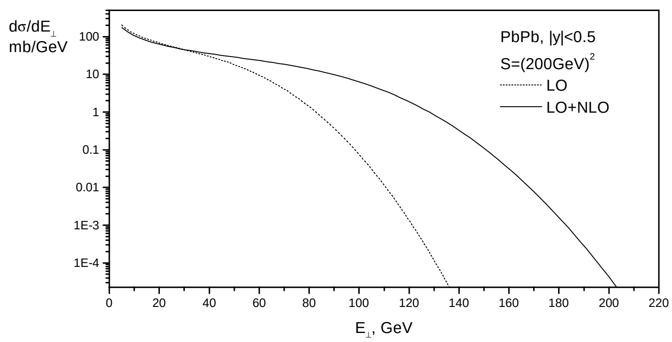

Fig. 3. NLO (solid line) and LO (dashed line) transverse-energy spectrum in a unit central rapidity window for PbPb collisions at RHIC energy $\sqrt{s} = 200$ GeV

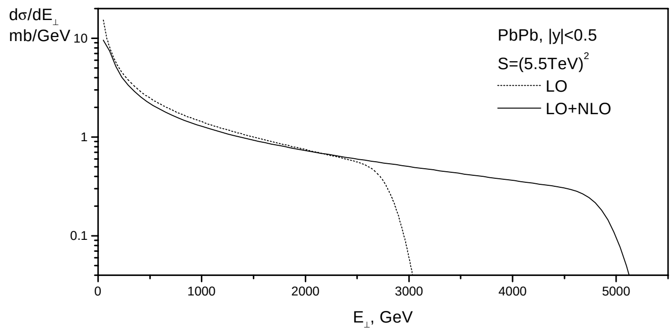

Fig. 4. NLO (solid line) and LO (dashed line) transverse-energy spectrum in a unit central rapidity window for PbPb collisions at LHC energy $\sqrt{s} = 5500$ GeV

NLO. At the same time, the height of transverse-energy distribution basically does not change. The above results show that to this order in perturbative QCD computation, the NLO prediction is an increased event-by-event-produced transverse energy, providing more favorable conditions for collective behavior of the produced gluon system, its thermalization, etc., as compared to the leading-order calculations.

4 Discussion

The results of the above-presented calculation of the next-to-leading order corrections to the transverse-energy flow into a unit rapidity interval in the central region show that the NLO corrections to the Born calculation of the transverse-energy distribution in pp collisions based on the lowest-order $2 \to 2$ scattering are substantial, so that the effective K factors are $K_{\mathrm{RHIC}} \sim 1.9$ and $K_{\mathrm{LHC}} \sim 2.1$, in accordance with the expected ones previously used in the literature [2,3].

The cross section of transverse-energy production in pp collisions serves as a basic building block in the geometrical Glauber model of nuclear collisions. Switching from LO to NLO cross section of $E_\perp$ production results in a substantial broadening of the minijet transverse-energy distribution in nuclear collision, providing more favorable conditions for subsequent collective effects characteristic for dense parton systems, such as quark–gluon plasma, to manifest themselves. We note that the form of this distribution does not change very much.

Because the effective K factors are substantial (although not overwhelmingly large), this makes it necessary to develop a more accurate treatment of minijet production. To achieve this goal, one has to solve two interrelated problems. Both are linked to the large value of the integrated minijet transverse-energy production cross section in the leading twist approximation, discussed in the second section, and the resulting conflict with unitarity at low minijet transverse energies at LHC, at least when large enough rapidity windows are considered.

First of all, one has to get a reliable estimate of the higher-order corrections to the hard blob responsible for $E_\perp$ production. This will most probably require resumming the perturbative contributions to all orders, because even if the hypothetical next-to-next-to-leading-order calculation reduces the K factor, the arising large cancellations would require further improvement of the accuracy of the calculation. Such a resummation program has been successfully utilized in the case of jet pair production at the kinematical threshold [29]. Unfortunately, it is not clear at present how to extend this program to the minijet production kinematics.

The second problem is even more difficult and is related to the necessity of a reliable computation of the nonlinear contributions to (mini)jet production, which are quite important at high energies in both hadron and nuclear collisions [2] and photoproduction [28]. The current approach to describing nonlinear effects is based on the *ad hoc* eikonal unitarization scheme; see, e.g., [2]. This scheme does not take into account the processes in which several nucleons are involved in transverse energy production. This problem was recently reanalyzed in [30] for pA collisions, showing in particular an interesting connection between the nonlinear effects in the structure functions and those in the spectrum of emitted gluons. An advanced analysis of the nonlinear effects for the example of nuclear effects in jet photoproduction [31] has revealed a number of interesting features that are possibly relevant for the computation of nonlinear effects in hadroproduction of jets. One of the most striking aspects of this treatment is that although diagrammatically, the nonlinear effects initially appear to be a superposition of the leading twist contributions, the final result appears to be a next-to-leading twist one, because of a delicate cancellation between various diagrams [31,32].

Various theoretical schemes possibly responsible for taming the growth of the minijet production cross sections were discussed. One of them is based on accounting for nonlinear effects in the quasi-classical approach [33, 12]. Here, at tree level, the nonperturbative lattice calculations of minijet production in nuclear collisions were performed in [13]. A more traditional treatment based on accounting for nonlinear effects in QCD evolution equations is described in the review [34].

Another related problem is a necessity of switching from the collinear to high-energy factorization when describing the minijet production at high energies; see, e.g., [35,12].

In summary, the computed NLO corrections to the minijet transverse energy production in hadron and nuclear collisions turned out to be substantial both for RHIC and LHC energies. Qualitatively, this enhances the energy production in the central region and significantly broadens the transverse-energy spectrum in nuclear collisions. However, much work is still ahead for the improvement of the accuracy of these predictions.

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